SM223 – Calculus III with Optimization
Assoc. Prof. Nelson Uhan

Fall 2017

Lesson 3. The Dot Product

1 Today...

- Definition and properties of the dot product
- Dot products and angles between vectors

2 The dot product

- We know how to multiply a vector by a scalar
- Can we multiply two vectors together? Yes!
- If $\vec{a} = \langle a_1, a_2, a_3 \rangle$ and $\vec{b} = \langle b_1, b_2, b_3 \rangle$, the **dot product** of $\vec{a}$ and $\vec{b}$ is

- Note that $\vec{a} \cdot \vec{b}$ is a scalar
- The dot product of vectors in $\mathbb{R}^2$ is defined similarly: if $\vec{a} = \langle a_1, a_2 \rangle$ and $\vec{b} = \langle b_1, b_2 \rangle$, then

Example 1.

a. $\langle -1, 7 \rangle \cdot \langle 6, 2 \rangle =$

b. $\langle 2, 4, 1 \rangle \cdot \langle -1, 3, 1 \rangle =$

c. $(-\vec{i} + 3\vec{k} + 4\vec{j}) \cdot (\vec{i} - 3\vec{k}) =$

- **Properties of the dot product**

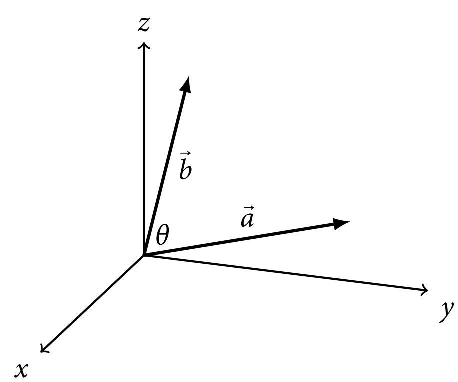
$$\vec{a} \cdot \vec{a} = |a|^2 \qquad (c\vec{a}) \cdot \vec{b} = c(\vec{a} \cdot \vec{b}) = \vec{a} \cdot (c\vec{b})$$
$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a} \qquad \vec{0} \cdot \vec{a} = 0$$
$$\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$$

- The dot product behaves very similarly to ordinary products of real numbers

Example 2. Show the first property above: $\vec{a} \cdot \vec{a} = |a|^2$.

3 Dot products and angles

- The **angle** θ between two vectors $\vec{a}$ and $\vec{b}$:



  - We always take the angle so that $0 \leq \theta \leq \pi$

- If $\vec{a}$ and $\vec{b}$ are scalar multiples of one another, we say that the vectors are **parallel**
  - If $\vec{a}$ and $\vec{b}$ are parallel, then $\theta =$

- If θ is the angle between vectors $\vec{a}$ and $\vec{b}$, then

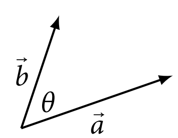
$\Rightarrow$ If θ is the angle between nonzero vectors $\vec{a}$ and $\vec{b}$, then

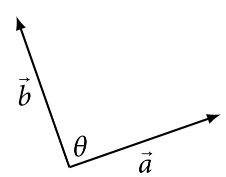
Example 3. Find the angle between vectors $\vec{a} = \langle 2, -1, 3 \rangle$ and $\vec{b} = \langle -3, 2, 5 \rangle$.

- Two nonzero vectors $\vec{a}$ and $\vec{b}$ are called **perpendicular** or **orthogonal** if the angle between them is $\theta = \pi/2$
- Suppose $\vec{a}$ and $\vec{b}$ are nonzero
  - If $\vec{a}$ and $\vec{b}$ are perpendicular, then $\vec{a} \cdot \vec{b} =$
  - If $\vec{a} \cdot \vec{b} = 0$, then $\cos\theta =$ and so $\theta =$

$\Rightarrow$ Two vectors $\vec{a}$ and $\vec{b}$ are orthogonal if and only if

Example 4. Show that $2\vec{i} - \vec{j} + 2\vec{k}$ is perpendicular to $5\vec{i} + 2\vec{j} - 4\vec{k}$.

- The dot product measures the extent to which $\vec{a}$ and $\vec{b}$ point in the same direction
  - 

  - 

  -